

Logic and Constraint Programming

2- Backtracking

A.A. 2021/2022



Lorenzo Rossi

lorenzo.rossi@unicam.it

University of Camerino

EXERCISE

» GRAPH COLOURING PROBLEM



What about central Italy?



Backtracking

BACKTRACKING

» BACKTRACKING SEARCH



A possible efficient and simple method.

- Variables are instantiated sequentially.
- After the variables of a constraint are instantiated, the constraint is checked
- If a (partial) instantiation violates a constraint, backtracking is performed to the most recently instantiated variable that still has alternative values

Backtracking eliminates a subspace from the Cartesian product of all variable domains.

Essentially a depth-first search variant.

BACKTRACKING

» EXAMPLE



Consider the following CSP:

- Variables:

A, B, C

- Domains:

$D(A) = D(B) =$

$D(C) = \{1, 2, 3\}$

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$A > B; B \neq C;$

$A \neq C;$

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How

represent it?

BACKTRACKING

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BACKTRACKING

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Consider the following CSP:

- Variables:

A, B, C

How we can represent the state space?

- Domains:

$D(A) = D(B) =$

$D(C) = \{1, 2, 3\}$

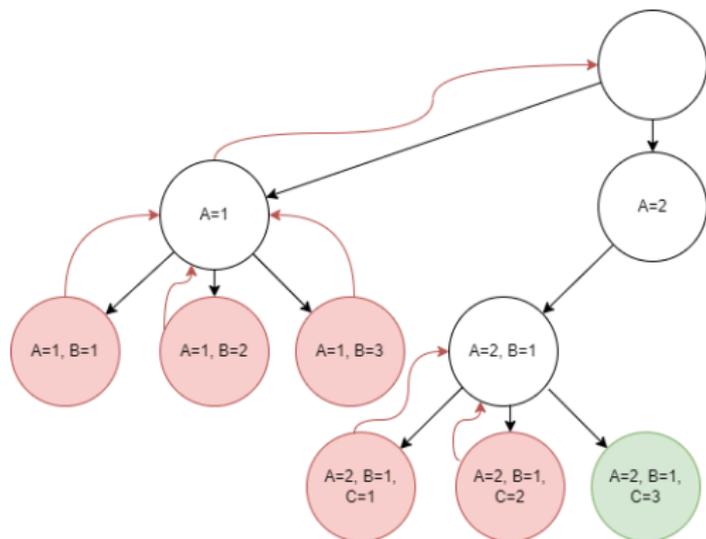
- Constraints:

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BACKTRACKING

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BACKTRACKING



General-purpose methods can give huge gains in speed:

- Which variable should be assigned next?
- In what order should its values be tried?
- Can we detect inevitable failure early?

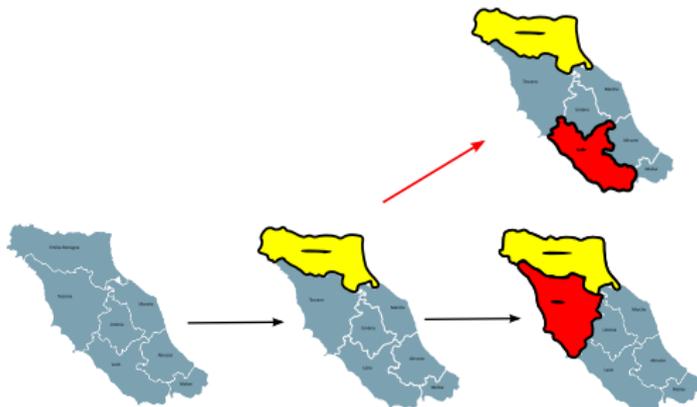
HEURISTICS

» VARIABLE ORDERING



Minimum remaining values heuristic

Pick the most constrained variable



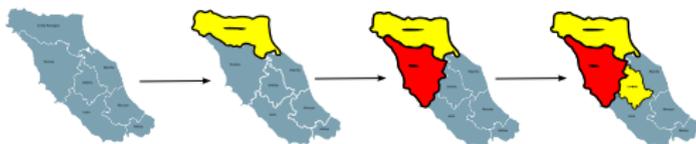
HEURISTICS

» VALUE SELECTION



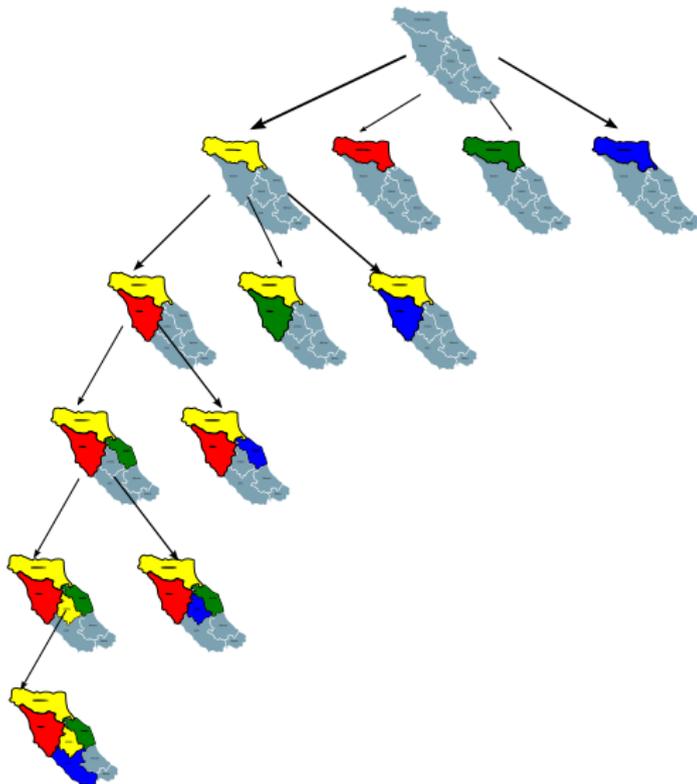
Least constraining value heuristic

the one that rules out the fewest values in the remaining variables



BACKTRACKING

» GRAPH COLOURING



FORWARD CHECKING



Forward checking

prevents assignments that guarantee later failure

- Keep track of remaining legal values for unassigned variables
- Terminate search when any variable has no legal values

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●	●	●	●	●	●	●
●	●	●	●	●	●	●
●	●	●	●	●	●	●
●	●	●	●	●	●	●
●	●	●	●	●	●	●